BRIN MATHEMATICS RESEARCH CENTER

Frontiers of Numerical PDEs

MAY 16 - 19, 2023

About the Workshop

This conference aims to bring together a diverse set of researchers in numerical analysis to present state-of-the-art algorithms and approximation techniques for fractional diffusion equations, geometric evolution problems, optimal transport, and related PDEs. It will provide a collegial atmosphere for productive discussion and interchange of ideas regarding new tools for studying the stability, approximation, and convergence of numerical methods to solve a diverse range of problems in numerical analysis and scientific computing. The conference will also celebrate Ricardo H. Nochetto's 70th birthday, and honor his contributions to all of the above areas.

Organizers

Andrea Bonito, Texas A&M University
Enrique Otarola, Universidad Tecnica Federico Santa Maria, Chile
Abner J. Salgado, University of Tennessee
Shawn W. Walker, Louisiana State University

Local Organizing Committee

Howard Elman, University of Maryland
Tobias von Petersdorff, University of Maryland

Plenary Speakers

Mark Ainsworth, Brown University
Zhiming Chen, Chinese Academy of Sciences
Bernardo Cockburn, University of Minnesota
Ronald DeVore, Texas A&M University
Guiseppe Savare, Bocconi University
James Sethian, UC Berkeley
Jinchao Xu, Pennsylvania State University

Invited Speakers

Georgios Akrivis, University of Ioannina
Eberhard Baensch, Friedrich-Alexander-Universität Erlangen-Nürnberg
Soeren Bartels, Albert Ludwigs University of Freiburg
Daniele Boffi, King Abdullah University of Science and Technology
Lucas Bouck, University of Maryland
Claudio Canuto, Politecnico di Torino
Long Chen, University of California Irvine
Georg Dolzmann, Universität Regensburg
Lucia Gastaldi, Università degli Studi di Brescia
Vivette Girault, Laboratoire Jacques-Louis Lions
Diane Guignard, University of Ottawa
Omar Lakkis, University of Sussex
Stig Larsson, Chalmers University of Technology
Charalambos Makridakis, University of Sussex
Pedro Morin, Universidad Nacional del Litoral
Maurizio Paolini, Università Cattolica del Sacro Cuore di Brescia
Rob Stevenson, University of Amsterdam
Andreas Veeser, Università degli Studi di Milano

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DEPARTMENT OF MATHEMATICS

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Workshop Overview

In the past decade, the numerical approximation of partial differential equations (PDEs) has been pushed into new and exciting application areas. Complex phenomena, with fine-scale structure, can be modeled by "generalized", fractional order differential equations. Many types of free boundary problems in science and engineering have a significant geometric component, such as curvature. Liquid crystal (LC) models combine both geometric and topological constraints with vector and tensor order parameters to yield a coarse-grained description of the physics of LC devices. Optimal transport, a major theory in analysis, has now become a practical tool with many applications. And modern adaptive algorithms are able to optimize and balance the computational effort to capture small scales without over-resolving the others.

This conference aims to bring together a diverse set of researchers in numerical analysis to present state-of-the-art algorithms and approximation techniques for the problems listed above. It will provide a collegial atmosphere for productive discussion and interchange of ideas regarding new tools for studying the stability, approximation, and convergence of numerical methods to solve a diverse range of problems in numerical analysis and scientific computing. The conference will also celebrate Ricardo H. Nochetto's 70th birthday, and honor his contributions to all of the above areas.

Partial support for this event was provided by the Institute for Physical Science and Technology at the University of Maryland.

Organizing committee

ANDREA BONITO, Texas A&M University

ENRIQUE OTAROLA, Universidad Tecnica Federico Santa Maria, Chile

ABNER J. SALGADO, University of Tennessee

SHAWN W. WALKER, Louisiana State University

HOWARD ELMAN, University of Maryland

TOBIAS VON PETERSDORFF, University of Maryland

Workshop Schedule

Tuesday, May 16, 2023

3:30 - 4:30 Jinchao Xu (Pennsylvania State University)
From finite element to deep learning

4:30 - 5:00 Coffee Break

5:00 - 5:30 Pedro Morin (Universidad Nacional del Litoral)
On approximation classes for adaptive time-stepping finite element methods

5:30 - 6:00 Long Chen (University of California, Irvine)
Finite Element Complexes

Wednesday, May 17, 2023

9:00 - 10:00	James Sethian (University of California, Berkeley) *Advances in Advancing Interfaces: The Mathematics of Manufacturing of Industrial Foams, Fluidic Devices, and Automobile Painting*
10:00 - 10:30	Coffee Break
10:30 - 11:00	Vivette Girault (Laboratoire Jacques-Louis Lions – virtual) *A discretization of the ill-posed Cauchy problem for the Laplace equation*
11:00 - 11:30	Stig Larsson (Chalmers University of Technology – virtual) *A priori and a posteriori error estimates for discontinuous Galerkin time-discrete methods via maximal regularity*
11:30 - 12:00	Eberhard Baensch (Friedrich-Alexander-Universität Erlangen-Nürnberg) *A thermodynamically consistent model for convective transport in nanofluids*
12:00 - 1:30	Lunch
1:30 - 2:30	Ronald DeVore (Texas A&M University) *Optimality in computation*
2:30 - 3:00	Coffee Break
3:00 - 3:30	Lucia Gastaldi (Università degli Studi di Brescia) *Virtual element approximation of eigenvalue problems*
3:30 - 4:00	Soeren Bartels (Albert Ludwigs University of Freiburg) *Error representation for variational problems and application to TV minimization*
4:00 - 4:30	Coffee Break
4:30 - 5:00	Charalambos Makridakis (IACM-FORTH Crete and University of Sussex) *Stability and convergence of machine learning algorithms*
5:00 - 5:30	Bernardo Cockburn (University of Minnesota – virtual) *Adaptivity for Hamilton-Jacobi equations*

Thursday, May 18, 2023

9:00 - 10:00	Zhiming Chen (Chinese Academy of Sciences – virtual) *Arbitrarily high order finite element methods for arbitrarily shaped domains with automatic mesh generation*
10:00 - 10:30	Coffee Break
10:30 - 11:00	Claudio Canuto (Politecnico di Torino – virtual) *Higher-order adaptive virtual element methods*
11:00 - 11:30	Rob Stevenson (University of Amsterdam – virtual) *A pollution-free ultra-weak FOSLS discretization of the Helmholtz equation*
11:30 - 12:00	Maurizio Paolini (Univ. Cattolica del Sacro Cuore di Brescia) *Soap films and partial wetting*
12:00 - 1:30	Lunch
1:30 - 2:30	Mark Ainsworth (Brown University) *High order mixed finite element approximation of Stokes equations*
2:30 - 3:00	Coffee Break
3:00 - 3:30	Diane Guignard (University of Ottawa) *Discontinuous Galerkin method for approximating the deformation of thin plates*
3:30 - 4:00	Omar Lakkis (University of Sussex – virtual) *Least-squares Galerkin recovery methods for fully nonlinear elliptic equations*
4:00 - 4:30	Coffee Break
4:30 - 5:00	Georg Dolzmann (Universität Regensburg) *Evolution of vector fields on flexible curves and surfaces*
5:00 - 5:30	Lucas Bouck (University of Maryland) *Finite element approximation of a membrane model for liquid crystal polymeric networks*

Friday, May 19, 2023

9:00 - 10:00	Giuseppe Savare (Bocconi University) *Evolution equations in spaces of probability measures*
10:00 - 10:30	Coffee Break
10:30 - 11:00	Andreas Veeser (Università degli Studi di Milano – virtual) *An adaptive finite element method with error-dominated oscillation*
11:00 - 11:30	Georgios Akrivis (University of Ioannina) *Discontinuous Galerkin time-stepping methods for nonlinear parabolic equations*
11:30 - 1:00	Lunch
1:00 - 2:00	Daniele Boffi (King Abdullah University of Science and Technology (KAUST)) *Approximation of parametric eigenvalue problems arising from partial differential equations*

Abstracts of talks

From finite element to deep learning

Jinchao Xu

Pennsylvania State University

Tuesday, May 16, 2023 @ 3:30 PM

In this talk, I will discuss how finite element method and theory can be used to understand, design and analyze deep learning models and training algorithms. In particular, I will present the following results. (1) ReLU neural networks are identical to linear finite element functions; (2) A convolutional neural network (CNN) can be obtained by a minor modification of a classic multigrid method (MgNet) (3) Convergence of gradient descent method for ReLU neural network can be analyzed by relating it to Jacobi iteration for a standard finite element stiffness matrix (4) A one dimensional finite element nodal basis function can be used as an effective activation function for CNN. (5) An efficient federated learning algorithm can be obtained and analyzed as a dual of some parallel subspace correction method; Furthermore, I will discuss and compare approximation and adaptivity properties of the finite element versus the finite neural method for numerical solution of partial differential equations.

On approximation classes for adaptive time-stepping finite element methods

Pedro Morin

Universidad Nacional del Litoral

Tuesday, May 16, 2023 @ 5:00 PM

Finite Element Complexes

Long Chen

University of California, Irvine

Tuesday, May 16, 2023 @ 5:30 PM

A Hilbert complex is a sequence of Hilbert spaces connected by a sequence of closed densely defined linear operators satisfying the property: the composition of two consecutive maps is zero. The most well-known example is the de Rham complex involving grad, curl, and div operators. A finite element complex is a discretization of a Hilbert complex by replacing infinite dimensional Hilbert spaces by finite dimensional subspaces based on a mesh of the domain. Usually inside each element of the mesh, polynomial spaces are used and suitable degree of freedoms are proposed to glue them to form a conforming subspace. The finite element de Rham complexes are well understood and can be derived from the framework Finite Element Exterior Calculus (FEEC).

In this talk, we will survey the construction of finite element complexes. We present finite element de Rham complex by a geometric decomposition approach. We then generalize the construction to smooth FE de Rham complexes and derive more complexes including the Hessian complex, the elasticity complex, and the divdiv complex by the Bernstein-Gelfand-Gelfand (BGG) construction. The constructed finite element complexes will have application in the numerical simulation of the biharmonic equation, the linear elasticity, the general relativity, and in general PDEs in Riemannian geometry etc.

This is a joint work with Xuehai Huang from Shanghai University of Finance and Economics.

Advances in Advancing Interfaces: The Mathematics of Manufacturing of Industrial Foams, Fluidic Devices, and Automobile Painting

James Sethian

University of California, Berkeley

Wednesday, May 17, 2023 @ 9:00 AM

Complex dynamics underlying industrial manufacturing depend in part on multiphase multiphysics, in which fluids and materials interact across orders of magnitude variations in time and space. In this talk, we will discuss the development and application of a host of numerical methods for these problems, including Level Set Methods, Voronoi Implicit Interface Methods, implicit adaptive representations, and multiphase discontinuous Galerkin Methods. Applications for industrial problems will include modeling how foams evolve, how electro-fluid jetting devices work, and the physics and dynamics of rotary bell spray painting across the automotive industry.

A discretization of the ill-posed Cauchy problem for the Laplace equation

Vivette Girault

Laboratoire Jacques-Louis Lions – virtual

Wednesday, May 17, 2023 @ 10:30 AM

A priori and a posteriori error estimates for discontinuous Galerkin time-discrete methods via maximal regularity

STIG LARSSON

Chalmers University of Technology – virtual

Wednesday, May 17, 2023 @ 11:00 AM

A thermodynamically consistent model for convective transport in nanofluids

EBERHARD BAENSCH

Friedrich-Alexander-Universität Erlangen-Nürnberg

Wednesday, May 17, 2023 @ 11:30 AM

Optimality in computation

RONALD DEVORE

Texas A&M University

Wednesday, May 17, 2023 @ 1:30 PM

Virtual element approximation of eigenvalue problems

Lucia Gastaldi

Università degli Studi di Brescia

Wednesday, May 17, 2023 @ 3:00 PM

The Virtual Elememnt Method (VEM) has been introduced about ten years ago to approximate the solution of several PDEs using polygonal/polyhedral meshes. An important feature of VEM is that suitable stabilizing forms, depending on appropriate parameters, have to be introduced in order to guarantee consistency and stability of the approximation. This is due to the virtual nature of the basis functions corresponding to the degrees of freedom at the interior of elements. In this talk I shall focus on the application of VEMs to eigenvalue problems and, in particular, on the effect of the stabilization parameters on the approximation of the eigenvalues. Indeed, it has been observed recently that the choice of the optimal parameters for the computation of the eigenvalues of the Laplace operator might not be an easy task. Here I present some new results on the approximation of the eigenvalue for the acoustic vibration problem and we show that in some cases there is no need of using any stabilization for the stiffness and mass matrix. These results have been obtained, in collaboration with Daniele Boffi, Francesca Gardini, Linda N. Alzaben and Andreas S. Dedner.

Error representation for variational problems and application to TV minimization

Soeren Bartels

Albert Ludwigs University of Freiburg

Wednesday, May 17, 2023 @ 3:30 PM

Controlling the approximation error of finite element discretizations of variational problems is challenging when these involve constraints or are nondifferentiable. In many cases the primal-dual gap provides an upper bound on relevant error quantities. We show how this concept can be used to find exact representations of approximation errors which are fully computable and give rise to local mesh refinement. In particular, Crouzeix-Raviart finite element discretizations provide numerical approximations of primal and dual formulations simultaneously via a solution procedure for the primal problem and a subsequent post-processing. In addition, for a model problem defined on functions of bounded variation the nonconforming method leads to higher convergence rates than corresponding conforming methods which are further improved by local mesh refinement. This is joint work with Alex Kaltenbach (U Freiburg).

Stability and convergence of machine learning algorithms

Charalambos Makridakis

IACM-FORTH Crete and University of Sussex

Wednesday, May 17, 2023 @ 4:30 PM

Motivated by tools in nonlinear PDEs we discuss new results on the stability and convergence of some widely used machine learning algorithms. In particular, we will consider methods approximating functions as well as solutions of elliptic and parabolic PDEs by neural networks.

Adaptivity for Hamilton-Jacobi equations

Bernardo Cockburn

University of Minnesota – virtual

Wednesday, May 17, 2023 @ 5:00 PM

Arbitrarily high order finite element methods for arbitrarily shaped domains with automatic mesh generation

Zhiming Chen

Chinese Academy of Sciences – virtual

Thursday, May 18, 2023 @ 9:00 AM

Higher-order adaptive virtual element methods

Claudio Canuto

Politecnico di Torino – virtual

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A pollution-free ultra-weak FOSLS discretization of the Helmholtz equation

Rob Stevenson

University of Amsterdam – virtual

Thursday, May 18, 2023 @ 11:00 AM

Soap films and partial wetting

Maurizio Paolini

Univ. Cattolica del Sacro Cuore di Brescia

Thursday, May 18, 2023 @ 11:30 AM

High order mixed finite element approximation of Stokes equations

Mark Ainsworth

Brown University

Thursday, May 18, 2023 @ 1:30 PM

We shall give an overview of recent work developing high order mixed finite element schemes for the approximation of Stokes equations. In particular, the methods are pressure robust and uniformly inf-sup stable in both the mesh-size and the polynomial degree and deliver exponential rates of convergence. The implementation and preconditioning of the elements will also be briefly mentioned.
This is based on work with Charles Parker - a graduate of the University of Maryland.

Discontinuous Galerkin method for approximating the deformation of thin plates

Diane Guignard

University of Ottawa

Thursday, May 18, 2023 @ 3:00 PM

We study the elastic behavior of prestrained and bilayer thin plates which can undergo large deformations and achieve non-trivial equilibrium shapes even without external forces or prescribed boundary conditions. These phenomena can be observed in nature or be manufactured. Being able to simulate the deformation of such plates can be beneficial for many engineering applications, for instance to develop micro-mechanical devices or to design climate-responsive architecture. We are mainly interested in the bending regime, namely when the three-dimensional hyper-elastic energy scales like the third power of the thickness of the plate. From a mathematical point of view, the dimensionally reduced problem consists of a fourth order minimization problem subject to a nonlinear and nonconvex metric constraint.
In this talk, we introduce a numerical method based on a discontinuous Galerkin finite element method for the space discretization and a discrete gradient flow for the energy minimization. We discuss the properties of the method and present numerical experiments showcasing the large variety of shapes that can be achieved.

Least-squares Galerkin recovery methods for fully nonlinear elliptic equations

Omar Lakkis

University of Sussex – virtual

Thursday, May 18, 2023 @ 3:30 PM

Evolution of vector fields on flexible curves and surfaces

Georg Dolzmann

Universität Regensburg

Thursday, May 18, 2023 @ 4:30 PM

In this lecture we discuss some recent progress on a model system consisting of a flexible surface and a vector field defined on the surface in the case in which an interaction between the vector field and the conformation of the surface is present. Recent approaches towards the existence of solutions will be reviewed and short time existence will be established.
The lecture is based on joint work with Christopher Brand (Regensburg), Julia Menzel (Regensburg) and Alessandra Pluda (Pisa)

Finite element approximation of a membrane model for liquid crystal polymeric networks

Lucas Bouck

University of Maryland

Thursday, May 18, 2023 @ 5:00 PM

Liquid crystal polymeric networks are materials where a nematic liquid crystal is coupled with a rubbery material. When actuated with heat or light, the interaction of the liquid crystal with the rubber creates complex shapes. Starting from the classical 3D trace formula energy of Bladon, Warner and Terentjev (1994), we derive a 2D membrane energy as the formal asymptotic limit of the 3D energy. The derivation is similar to derivations in Ozenda, Sonnet, and Virga (2020) and Cirak et. al. (2014). We characterize the zero energy deformations and prove that the energy lacks certain convexity properties. We propose a finite element method to discretize the problem. To address the lack of convexity of the membrane energy, we regularize with a term that mimics a higher order bending energy. We prove that minimizers of the discrete energy converge to minimizers of the continuous energy. For minimizing the discrete problem, we employ a nonlinear gradient flow scheme, which is energy stable. Additionally, we present computations showing the geometric effects that arise from liquid crystal defects. Computations of configurations from nonisometric origami are also presented.

Evolution equations in spaces of probability measures

Giuseppe Savare

Bocconi University

Friday, May 19, 2023 @ 9:00 AM

The space of Borel probability measures (on a Euclidean or Hilbert space) endowed with the L^2 Kantorovich-Rubinstein-Wasserstein metric has a kind of non-smooth infinitesimal Riemannian structure, which is useful for studying evolution problems and mean-field limits of dynamical particle systems. These are typically described by a continuity equation where the velocity field depends on the evolving measures.

Despite the fact that the geometry of the space is strongly influenced by non-smooth curvature effects, tools from the theory of gradient flows and contraction semigroups in Hilbert-Banach spaces can be used to characterize solutions and derive optimal error estimates for implicit and explicit Euler schemes.

We give a brief overview of the problems and theoretical results concerning flows generated by Wasserstein gradients of displacement convex functions and by more general probability vector fields satisfying a suitable dissipativity condition.

An adaptive finite element method with error-dominated oscillation

Andreas Veeser

Università degli Studi di Milano – virtual

Friday, May 19, 2023 @ 10:30 AM

Employing Crouzeix-Raviart elements and bisection, we consider the adaptive solution of the Poisson equation for certain right-hand sides. Square-integrable load functions are in particular covered. Basing upon a sharpened a posteriori analysis and conforming adaptive tree approximation by Binev/Fierro/Veeser, we propose an adaptive algorithm that generates near-best approximations in a nonlinear sense. In other words, the error of any generated approximate solution is bounded in terms of the best error achievable by means of a comparable number of bisections and some multiplicative constant. This is a joint work with Francesca Fierro (Milano).

Discontinuous Galerkin time-stepping methods for nonlinear parabolic equations

Georgios Akrivis

University of Ioannina

Friday, May 19, 2023 @ 11:00 AM

We consider the discretization of nonlinear parabolic equations of the form $u_t = \nabla \cdot f(\nabla u, u)$, with f a smooth flux function satisfying a local ellipticity condition, by discontinuous Galerkin time-stepping methods. We use maximal regularity properties of discontinuous Galerkin methods for linear nonautonomous parabolic equations and establish error estimates.

Approximation of parametric eigenvalue problems arising from partial differential equations

Daniele Boffi

King Abdullah University of Science and Technology (KAUST)

Friday, May 19, 2023 @ 1:00 PM

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

Andrea Bonito, Texas A&M University
Andreas Veeser, Università degli Studi di Milano – virtual
Antonio de Rosa, University of Maryland
Bernardo Cockburn, University of Minnesota – virtual
Celine Torres, University of Maryland
Charalambos Makridakis, IACM-FORTH Crete and University of Sussex
Claudio Canuto, Politecnico di Torino – virtual
Daniele Boffi, King Abdullah University of Science and Technology (KAUST)
Diane Guignard, University of Ottawa
Dionetis Margetis, University of Maryland
Doron Levy, University of Maryland
Eberhard Baensch, Friedrich-Alexander-Universität Erlangen-Nürnberg
Enrique Otarola, Universidad Tecnica Federico Santa Maria, Chile
Georg Dolzmann, Universität Regensburg
Georgios Akrivis, University of Ioannina
Giuseppe Savare, Bocconi University
Guillaume Bonnet, University of Maryland
Gunay Dogan, NIST
Harbir Antil, George Mason University
Howard Elman, University of Maryland
James Sethian, University of California, Berkeley
Jinchao Xu, Pennsylvania State University
Konstantina Trivisa, University of Maryland
Long Chen, University of California, Irvine
Lucas Bouck, University of Maryland
Lucia Gastaldi, Università degli Studi di Brescia
Mark Ainsworth, Brown University
Maurizio Paolini, Univ. Cattolica del Sacro Cuore di Brescia
Omar Lakkis, University of Sussex – virtual
Patrick Sodre, Intelligent Automation
Pedro Morin, Universidad Nacional del Litoral
Rob Stevenson, University of Amsterdam – virtual
Ronald DeVore, Texas A&M University
Shawn W. Walker, Louisiana State University
Soeren Bartels, Albert Ludwigs University of Freiburg

Stig Larsson, Chalmers University of Technology – virtual
Stu Antman, University of Maryland
Tobias von Petersdorff, University of Maryland
Vivette Girault, Laboratoire Jacques-Louis Lions – virtual
Zhiming Chen, Chinese Academy of Sciences – virtual