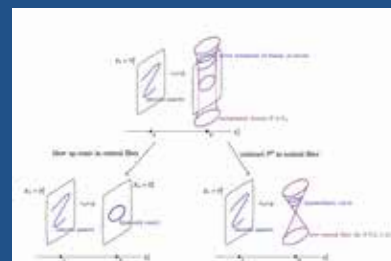
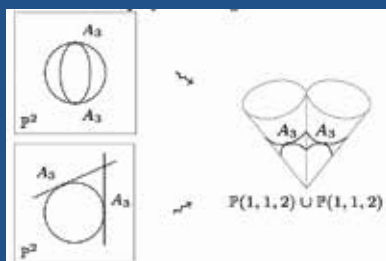


PERSPECTIVES ON MODULI IN ALGEBRAIC GEOMETRY

JANUARY 27-31, 2025



SPEAKERS

Valery Alexeev, University of Georgia
Harold Blum, University of Utah
Chiara Damiolini, University of Texas at Austin
Dan Halpern-Leistner, Cornell University
Giovanni Inchiostro, University of Washington
Yunfeng Jiang, University of Kansas
Anne-Sophie Kaloghiros, Brunel University London
János Kollár, Princeton University
Sándor Kovács, University of Washington
Radu Laza, Stony Brook University
Donggun Lee, IBS Center for Complex Geometry
Yongnam Lee, KAIST/IBS Center for Complex Geometry
Sam Molcho, Sapienza University of Rome
Han-Bom Moon, Fordham University
Julie Rana, Lawrence University
Giancarlo Urzua, Universidad Católica de Chile
Jeremy Usatine, Florida State University
Chenyang Xu, Princeton University
Aline Zanardini, École Polytechnique Fédérale de Lausanne
Ziquan Zhuang, Johns Hopkins University

The central problem in algebraic geometry is to classify algebraic varieties or related geometric structures. Algebraic varieties are often parametrized by certain moduli spaces and the geometry of these moduli spaces encodes the ways of continuously deforming the varieties. Answering the classification question often boils down to understanding the geometry of these moduli spaces. In the past few years, new tools have been developed in moduli theory but there are still many open questions, especially in higher dimensions. This workshop will bring a diverse group of researchers in moduli theory to discuss new developments and perspectives in the field.

ORGANIZERS

Kenny Ascher, UC Irvine
Dori Bejleri, University of Maryland
Kristin DeVleming, UC San Diego
Junyan Zhao, University of Maryland



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Schedule at a Glance

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00					
9:00	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
10:00	Kaloghiros	Molcho	Kollar	Alexeev	Usatine
11:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
12:00	Zhuang	Blum	Blum	Jiang	Inchiostro
13:00	Break	Laza	Break	Break	
14:00	Zanardini	Lunch (provided)	Halpern-Leistner	Urzua	Xu
15:00	Lunch (provided)		Lunch (on your own)	Lunch (provided)	Lunch (on your own)
16:00	Rana	Moon	Free afternoon in DC	Lee	
17:00	Coffee Break	Coffee Break		Coffee Break	
18:00	Kovacs	Lee		Damiolini	
	High Tea				

Workshop Overview

The central problem in algebraic geometry is to classify algebraic varieties or related geometric structures. Algebraic varieties are often parametrized by certain moduli spaces and the geometry of these moduli spaces encodes the ways of continuously deforming the varieties. Answering the classification question often boils down to understanding the geometry of these moduli spaces. In the past few years, powerful new tools have been developed in moduli theory but there are still many open questions, especially in higher dimensions. This workshop will bring together a diverse group of researchers in moduli theory to discuss new developments and perspectives in the field.

Organizing committee

DORI BEJLERI University of Maryland

KENNY ASCHER, UC Irvine

KRISTIN DEVLEMING, UC San Diego

JUNYAN ZHAO, University of Maryland

Workshop Schedule

MONDAY, JANUARY 27, 2025

- 8:30 - 8:50 BREAKFAST
- 8:50 - 9:00 DORON LEVY (University of Maryland/Director, Brin MRC)
Opening
- 9:00 - 10:00 ANNE-SOPHIE KALOGHIROS (Brunel University London)
Wall Crossing for K -Moduli of Fano Threefolds
- 10:00 - 10:30 COFFEE BREAK
- 10:30 - 11:30 ZIQUAN ZHUANG (Johns Hopkins University)
Boundedness of Singularities and Discreteness of Local Volumes
- 11:30 - 12:00 BREAK
- 12:00 - 1:00 ALINE ZANARDINI (Ecole Polytechnique Federale de Lausanne)
A Tale of Three GIT Problems
- 1:00 - 2:30 LUNCH (PROVIDED)
- 2:30 - 3:30 JULIE RANA (Lawrence University)
Noether-Lefschetz Loci for Families of Threefolds
- 3:30 - 4:00 COFFEE BREAK
- 4:00 - 5:00 SANDOR KOVACS (University of Washington)
KSB Stability is Automatic in Codimension 3
- 5:00 - 6:00 HIGH TEA

TUESDAY, JANUARY 28, 2025

8:30 - 9:00 BREAKFAST

9:00 - 10:00 SAM MOLCHO (Sapienza University of Rome)
Logarithmic Enumerative Geometry

10:00 - 11:00 COFFEE BREAK

11:00 - 12:00 RADU LAZA (Stony Brook University)
Hyper-Kähler Manifolds of Kummer Type

12:00 - 2:00 LUNCH (PROVIDED)

2:00 - 3:00 HAN-BOM MOON (Fordham University)
Ulrich Bundles on Intersections of Quadrics

3:00 - 4:00 COFFEE BREAK

4:00 - 5:00 YONGNAM LEE (KAIST/IBS Center for Complex Geometry)
Compact Moduli of Elliptic Surfaces with a Multiple Fiber

WEDNESDAY, JANUARY 29, 2025

- 8:30 - 9:00 BREAKFAST
- 9:00 - 10:00 JÁNOS KOLLÁR (Princeton University)
Néron models and minimal models
- 10:00 - 10:30 COFFEE BREAK
- 10:30 - 11:30 HAROLD BLUM (University of Utah)
Moduli of Boundary Polarized CY Surface Pairs
- 11:30 - 12:00 BREAK
- 12:00 - 1:00 DAN HALPERN-LEISTNER (Cornell University)
- 1:00 - 1:15 GROUP PHOTO
- 1:15 - 2:30 LUNCH (ON YOUR OWN)
- 2:30 - 7:00 FREE AFTERNOON IN DC

THURSDAY, JANUARY 30, 2025

- 8:30 - 9:00 BREAKFAST
- 9:00 - 10:00 VALERY ALEXEEV (University of Georgia)
Compact Moduli of Log Calabi-Yau Hyperplane Arrangements and of Calabi-Yau 3-Folds
- 10:00 - 10:30 COFFEE BREAK
- 10:30 - 11:30 YUNFENG JIANG (University of Kansas)
Equivariant Smoothing of Surface Singularities and Construction of Virtual Class on KSBA Spaces
- 11:30 - 12:00 BREAK
- 12:00 - 1:00 GIANCARLO URZUA (Universidad Catolica de Chile)
Classification of Horikawa Surfaces with T-Singularities
- 1:00 - 2:30 LUNCH (PROVIDED)
- 2:30 - 3:30 DONGGUN LEE (IBS Center for Complex Geometry)
Cohomology of the Hacking Moduli Space of Quartic Plane Curves
- 3:30 - 4:00 COFFEE BREAK
- 4:00 - 5:00 CHIARA DAMIOLINI (University of Texas at Austin)
Line Bundles on the Moduli of Parahoric Bundles
- 6:30 - 8:30 CONFERENCE DINNER

FRIDAY, JANUARY 31, 2025

- 8:30 - 9:00 BREAKFAST
- 9:00 - 10:00 JEREMY USATINE (Florida State University)
Stringy Hodge Numbers and Crepant Resolutions via Stacks
- 10:00 - 10:30 COFFEE BREAK
- 10:30 - 11:30 GIOVANNI INCHIOSTRO (University of Washington)
Stable Maps to Quotient Stacks with a Properly Stable Point
- 11:30 - 11:45 SHORT BREAK
- 11:45 - 12:45 CHENYANG XU (Princeton University)
Irreducible Symplectic Varieties with a Large Second Betti Number
- 12:45 - 12:50 WORKSHOP CLOSING
- 12:50 - 2:20 LUNCH (ON YOUR OWN)

Abstracts of talks

Wall Crossing for K-Moduli of Fano Threefolds

ANNE-SOPHIE KALOGHIROS

Brunel University London

Monday, January 27, 2025 @ 9:00 AM

I will discuss joint work with Ivan Cheltsov, Maksym Fedorchuk and Kento Fujita. In this talk, I will describe the component of the K-moduli space of smoothable Fano threefolds of anticanonical degree 24 associated to the deformation family 4.1 in the classification due to Mori and Mukai. Smooth Fano threefolds in this family are hypersurfaces of multi degree $(1,1,1,1)$ in $(P^1)^4$ (a product of four copies of P^1) and are K-polystable; I will describe singular K-polystable degenerations of these. I will relate this K-moduli space to K-moduli spaces of pairs $((P^1)^4, cX)$ where X is a Fano 3-fold in family 4.1, as c varies $0 < c < 2$, and discuss wall crossing for these.

Boundedness of Singularities and Discreteness of Local Volumes

ZIQUAN ZHUANG

Johns Hopkins University

Monday, January 27, 2025 @ 10:30 AM

The local volume of a Kawamata log terminal (klt) singularity is an invariant that plays a central role in the local theory of K-stability. By the stable degeneration theorem, every klt singularity has a volume preserving degeneration to a K-semistable Fano cone singularity. I will talk about a joint work with Chenyang Xu on the boundedness of Fano cone singularities when the volume is bounded away from zero. This implies that local volumes only accumulate around zero in any given dimension.

A Tale of Three GIT Problems

ALINE ZANARDINI

Ecole Polytechnique Federale de Lausanne

Monday, January 27, 2025 @ 12:00 PM

In this talk, I will report on work in progress joint with M. Hattori and T. Papazachariou concerning a classical cycle of correspondences among (certain) nets of quadrics in \mathbb{P}^3 , nets of cubics in \mathbb{P}^2 , and (smooth) plane curves of degree four. I will explain how, by extending these correspondences, one can obtain precise links among the three corresponding GIT problems.

Noether-Lefschetz Loci for Families of Threefolds

JULIE RANA

Lawrence University

Monday, January 27, 2025 @ 2:30 PM

For a fixed threefold X and very ample line bundle H , the Noether-Lefschetz locus parametrizes surfaces in X which are linearly equivalent to H and have Picard rank greater than that of X . We discuss the behavior of special components of the Noether-Lefschetz locus as we deform the pair (X, H) , particularly when X is (singular) Fano. This is joint work with A. Grassi and K. DeVleming.

KSB Stability is Automatic in Codimension 3

SANDOR KOVACS

University of Washington

Monday, January 27, 2025 @ 4:00 PM

In this talk I will first review KSB/A stability, especially their local version and then discuss joint work with Janos Kollar, showing that it is enough to check these conditions, including flatness, up to codimension 2. This implies that we have a very good understanding of this stability condition in general, because local KSB-stability is trivial at codimension 1 points, and quite well understood at codimension 2 points, since we have a complete classification of 2-dimensional slc singularities.

Logarithmic Enumerative Geometry

SAM MOLCHO

Sapienza University of Rome

Tuesday, January 28, 2025 @ 9:00 AM

In recent years, techniques from logarithmic geometry have been used to construct or study compactifications of several fundamental moduli spaces, such as moduli spaces of relative stable maps, line bundles on curves, and abelian varieties. A study of the enumerative geometry of these moduli spaces that truly interacts with their logarithmic nature however requires an enhancement of traditional intersection theory, called logarithmic intersection theory. In this talk, I will discuss the main points of this theory and survey some of the key new results that have come out of these considerations.

Hyper-Kähler Manifolds of Kummer Type

RADU LAZA

Stony Brook University

Tuesday, January 28, 2025 @ 11:00 AM

We say that a hyper-Kähler manifold is of Kummer type if it is birational to a quotient of an abelian variety. In this talk, I will explore the intriguing possibility that, up to deformations and birational transformations, all the hyper-Kähler manifolds are, in fact, of Kummer type. One case that we understand (almost) completely is the Lagrangian case. This latter part is joint work with Yoon-Joo Kim and Oliver Martin.

Ulrich Bundles on Intersections of Quadrics

HAN-BOM MOON

Fordham University

Tuesday, January 28, 2025 @ 2:00 PM

An Ulrich bundle is a vector bundle with very strong cohomology vanishing conditions. Eisenbud and Schreyer conjectured that every smooth projective variety possesses an Ulrich bundle. Despite many results on low dimensional varieties and special varieties, the general existence is unknown. In this talk, I will describe recent work in progress with Kyoung-Seog Lee and Jiwan Jeong on the construction of Ulrich bundles on an intersection of quadrics.

Compact Moduli of Elliptic Surfaces with a Multiple Fiber

YONGNAM LEE

KAIST/IBS Center for Complex Geometry

Tuesday, January 28, 2025 @ 4:00 PM

Motivated by Miranda and Ascher-Bejleri's works on compactification of moduli space of rational elliptic surfaces with a section, we study to construct compact moduli space of elliptic surfaces with a multiple fiber. Particular emphasis is placed on the study of rational elliptic surfaces without section and Dolgachev surfaces. The main approach to understanding limit surfaces is \mathbb{Q} -Gorenstein smoothing of slc surfaces. This is a joint work with Donggun Lee.

Néron models and minimal models

JÁNOS KOLLÁR

Princeton University

Wednesday, January 29, 2025 @ 9:00 AM

The lecture will discuss the automorphism groups of affine surfaces whose compactification is a $\log K3$ surface. As an application, we prove a conjecture of Mordell and Schinzel for integral points on cubic surfaces.

Moduli of Boundary Polarized CY Surface Pairs

HAROLD BLUM

University of Utah

Wednesday, January 29, 2025 @ 10:30 AM

While the theories of KSBA stability and K-stability have been successful in constructing compact moduli spaces of canonically polarized varieties and Fano varieties, respectively, the case of K-trivial varieties remains less well understood. I will discuss a new approach to this problem in the case of Calabi-Yau pairs (X, D) , where D is ample, in which we consider all semi-log-canonical degenerations. One challenge of this approach is that the set of such degenerations is unbounded and so the stack we work with is not finite type. Nevertheless, in the case of surface pairs, we construct a projective moduli space on which the Hodge line bundle is ample. This is based on joint work with Yuchen Liu that builds on previous work with Ascher, Bejleri, DeVleming, Inchiostro, Liu, and Wang.

TBD

DAN HALPERN-LEISTNER

Cornell University

Wednesday, January 29, 2025 @ 12:00 PM

Compact Moduli of Log Calabi-Yau Hyperplane Arrangements and of Calabi-Yau 3-Folds

VALERY ALEXEEV

University of Georgia

Thursday, January 30, 2025 @ 9:00 AM

Typically, the KSBA moduli spaces of log Calabi-Yau pairs $(X, B + \epsilon R)$ with $K_X + B \equiv 0$, ample R and small ϵ , are in some sense "toric" (see e.g. Losev-Manin spaces, moduli of abelian varieties, K3 surfaces, Enriques surfaces, anticanonical pairs). We exhibit a new instance of this phenomenon, where the moduli spaces are not toric but the stable varieties appearing on the boundary are. Let $0 < b_i < 1$, $i = 1, \dots, n$ be rational numbers adding up to r and consider the KSBA compactification of the moduli space of weighted hyperplane arrangements $(P^{r-1}, \sum(b_i + \epsilon_i)B_i)$. We prove that varieties appearing on the boundary are stable toric varieties given by explicit polyhedral tilings, which we describe. As an application, we explicitly describe a geometric compactification of a particular full-dimensional moduli space of Calabi-Yau 3-folds, giving an irreducible component in the moduli of all Calabi-Yau 3-folds. This is joint work with Xian Wu.

Equivariant Smoothing of Surface Singularities and Construction of Virtual Class on KSBA Spaces

YUNFENG JIANG

University of Kansas

Thursday, January 30, 2025 @ 10:30 AM

It is well-known that semi-log-canonical (slc) singularities of log general type varieties are the worst singularities in the Kollár-Shepherd-Barron-Alexeev (KSBA) compactification of moduli space of log general type varieties. In the surface case, the slc singularities were classified. Except the locally complete intersection (lci) singularities, the only slc surface singularities in the index one cover of an slc surface are simple elliptic singularities, cusp and degenerate cusp singularities. The smoothing of such singularities had been studied for a long time. In this talk we study the equivariant smoothing of such singularities by lci covers, and classify when such singularities admit equivariant smoothing of the same type lci singularities. The study of the equivariant smoothing is motivated by the construction of virtual fundamental class on KSBA spaces.

Classification of Horikawa Surfaces with T-Singularities

GIANCARLO URZUA

Universidad Catolica de Chile

Thursday, January 30, 2025 @ 12:00 PM

Horikawa surfaces are nonsingular minimal complex surfaces of general type with $K^2 = 2p_g - 4$. They were classified by E. Horikawa in the 70s. An interest in degenerations of Horikawa surfaces arises because of a famous problem proposed by Horikawa in 1976 about their diffeomorphism type when $K^2 = 16t$. Particularly interesting for this problem is to know about degenerations with only T-singularities. Until now there was only one example of such degenerations due to Y. Lee and J. Park 2011 (found by Fintushel-Stern 1996 as smooth 4-manifolds via rational blow-down surgery), we call them Lee-Park examples. Together with Jaime Negrete and Vicente Monreal, in this recent pre-print <https://arxiv.org/pdf/2410.02943> we classify all Horikawa surfaces with T-singularities, and we prove that for $p_g \geq 10$ the only smoothable surfaces are Lee-Park examples (and for just one component when there are two). In this classification we introduce a new family of singular surfaces, which we call small surfaces. Horikawa T-surfaces have 8 families of small surfaces, and so it proposes new diffeomorphism questions via rational blowdown. The novel techniques in our pre-print allow us to classify T-surfaces with $K^2 = 2p_g - 3$, e.g. quintics and I-surfaces. My talk will be about the various ideas behind the proofs.

Cohomology of the Hacking Moduli Space of Quartic Plane Curves

DONGGUN LEE

IBS Center for Complex Geometry

Thursday, January 30, 2025 @ 2:30 PM

In the early 2000s, Hacking introduced a compact moduli space of plane curves of a given degree. This is one of the first examples of compact moduli spaces of higher-dimensional pairs that generalize the moduli spaces of pointed (rational) curves. Despite its significant interest, the cohomology of this space has remained unexplored. Recent developments in moduli theory allow us to relate the Hacking moduli space to its GIT model through wall crossings of stability conditions. Building upon this, we compute the cohomology of the moduli space in the case of plane quartics. This is based on joint work in progress with Kenny Ascher.

Line Bundles on the Moduli of Parahoric Bundles

CHIARA DAMIOLINI

University of Texas at Austin

Thursday, January 30, 2025 @ 4:00 PM

Moduli spaces of vector bundles over a curve X have been a central focus in algebraic geometry and adjacent fields. A natural generalization of these spaces can be obtained by replacing vector bundles with G -bundles, for G an algebraic group. In this talk I focus on the case when G takes the form of a parahoric Bruhat-Tits group and discuss how, using representation-theoretical methods, one can describe the Picard group of Bun_G , the moduli space of G -bundles. Specifically, I show how twisted conformal blocks can be used to detect when line bundles on an appropriate flag variety descend to Bun_G . This is based on joint work with J. Hong.

Stringy Hodge Numbers and Crepant Resolutions via Stacks

JEREMY USATINE

Florida State University

Friday, January 31, 2025 @ 9:00 AM

Consider an invariant that behaves nicely for smooth varieties, such as Euler number, Betti numbers, or Hodge numbers. Suppose we want a version of this invariant for singular varieties that sees interesting information about the singularities. I will discuss how this naturally leads to the notion of crepant resolutions of singularities. However, crepant resolutions by smooth varieties are rare in practice. I will discuss joint work with M. Satriano in which we show that crepant resolutions actually exist in broad generality, as long as one is willing to consider algebraic stacks. Specifically, a variety admits a crepant resolution by a smooth algebraic stack if and only if it has log-terminal singularities. I will also discuss our new formula that expresses Batyrev's stringy Hodge numbers in terms of these stacky crepant resolutions.

Stable Maps to Quotient Stacks with a Properly Stable Point

GIOVANNI INCHIOSTRO

University of Washington

Friday, January 31, 2025 @ 10:30 AM

Given the construction of a moduli space of curves, it is natural to ask if one can construct a moduli space of pairs consisting of a curve C together with some extra data. When this extra data is parametrized by a moduli space M , this corresponds to constructing a moduli space of maps from curves to M . I will present a compactification of the moduli space of maps to certain moduli spaces M , via the example of M being the GIT moduli space of binary forms of degree $2n$. This leads to a compactification of the moduli space of fibrations $f : (X, D) \rightarrow C$, where each fiber of f is the projective line with a GIT-semistable configuration of $2n$ points. This is a joint work with Andrea Di Lorenzo.

Irreducible Symplectic Varieties with a Large Second Betti Number

CHENYANG XU

Princeton University

Friday, January 31, 2025 @ 11:45 AM

We show that the Lagrangian fibration constructed by Iiiev-Manivel using intermediate Jacobians of cubic fivefolds containing a fixed cubic fourfold, admits a compactification as a terminal \mathbb{Q} -factorial irreducible symplectic varieties. As far as I know, besides OG10, this is only the second family of irreducible symplectic varieties with the second Betti number at least 24. Joint work with Yuchen Liu and Zhiyu Liu.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

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NATHAN CHEN, Harvard University
ZHENGKAI PAN, Harvard University